Grade 7 Integer Subtraction

7.N.6	
Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically.	1. Explain, using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is zero. [NOT DEVELOPED - already developed in integer addition lesson] 2. Illustrate, using a number line, the results of adding or subtracting negative and positive integers (e.g., a move in one direction followed by an equivalent move in the opposite direction results in no net change in position). [ONLY FOR SUBTRACTION] 3. Add two integers using concrete materials or pictorial representations and record the process symbolically. [NOT DEVELOPED] 4. Subtract two integers using concrete materials or pictorial representations and record the process symbolically. [DEVELOPED] 5. Solve a problem involving the addition and subtraction of integers. [ONLY FOR SUBTRACTION]

Clarification of the outcome:

- ✦ This outcome is unpacked into two parts: addition and subtraction. Understanding subtraction depends on understanding addition. For this reason, addition should be taught first, in a lesson separate from subtraction. This lesson assumes that addition has already been developed.

Required close-to-at-hand prior knowledge:

- ✤ Understand what integers are, why they are useful, and how to compare them (bigger . . .)
- ✤ Understand integer addition.
- ✤ Understand opposites.
- ✤ Understand compensation thinking (e.g. to add 9, you can add 10, and subtract 1 to compensate).

Note:

Teaching students to subtract integers in a mechanical way is straightforward. You show them what to do and then provide lots of practice. Teaching students to understand integer subtraction is not straightforward and more difficult than teaching students to understand integer addition. This is partly because the models that can be used to teach integer addition do not necessarily work well for integer subtraction. For example, the gaining and losing model can help students understand integer addition. For this model, -5 + (-3) = -8 can be interpreted as I owe $5 to Joe and I owe $3 to Harry. When I combine debts, my total debt is $8. When owing and having is used as a context with the 'take away' meaning of subtraction, matters can get strange. Why? Consider the expression '-2 - (-3) = 1'. The interpretation, "*I owe $2 and when a debt of $3 is removed from me, I have $1 in my pocket*.", is suspect. A nice way to make money, but the mathematics does not make sense to many grade 7 students.

One popular approach to teaching integer subtraction involves using two-colour counters (see Grade 7 Integer addition). For example, to teach 3 - (-4), teachers have students represent 3 with three positive counters and -4 with four negative counters. Students are told to flip over the four negative counters (because they are being subtracted) and then add to get the answer. The seven positive counters represent the correct answer of 7, but the whole affair is somewhat magical. What teachers are doing is not much different from the good old days when students were told to change the sign and add. This time they are told to flip over the counters and add. The critical question of ‘<u>why is flipping over the counters equivalent to subtracting’</u> needs to be addressed if the “flip over” trick is to make sense to students.

Another popular approach to teaching integer subtraction involves using a number line. For example, to teach 3 - (-4), teachers have students start at +3 on the number line and then take 4 steps to the right (the other way: the opposite of -4), ending up at +7. This is the correct answer but the whole affair is as magical as for the two-colour counters model above. The critical question of ‘<u>why is walking the other way equivalent to subtracting’</u> needs to be addressed if the “walk the other way” trick is to make sense to students.

Another issue with teaching integer subtraction is the multiple meanings of ‘-‘ and the sloppy language attached to it. The dash (‘-‘) can represent a position on a number line (e. g. negative 7), the operation of subtraction, the opposite of [e.g. -(-4) means the opposite of negative 4], and a change (e.g. –7 can mean a decrease of 7). This is the main reason that the development of integer subtraction here begins with bracket notation (it distinguishes integer from operation).

As for language, consider the expression -(-2) – (-4). Teachers often translate the mathematical symbols present by saying: “minus minus 2 minus minus 4”. This confusing language does not help students distinguish between the use of ‘-‘ as a position on a number line, the arithmetic operation of subtraction, and the idea of an opposite. Preferable language could be “the opposite of minus 2 subtract minus 4”. In other words, it is important to use appropriate language that clearly separates the arithmetic operation called subtraction from the numbers that are being subtracted.

SET SCENE stage

Provide students with the following problem.

> The temperature at 9 AM was -5 Celsius. The temperature at 4 PM was -1 Celsius. What was the change in temperature?

The problem task to present to students:

Organize students into groups. Ask them to answer the question, using any method they like.

Encourage them to draw the situation on a thermometer (a vertical number line). Provide two-colour counters for students who might want to use them.

Comments:

The purpose of the task is present a situation that can be used to introduce integer subtraction in a context that should be familiar to students.

Note:

One issue with understanding integer arithmetic is the notation used. The adult world uses flat notation [e.g.: -2 + 7 = 5.] The teaching issue with this notation is that '+' and '-' can refer to an arithmetic operation (add or subtract) or to a number (a positive or negative integer). This can easily result in student confusion. Two notations specifically intended for teaching purposes have been used:

- Raised notation [e.g: $^{+}2 + {}^{-}3 = {}^{-}1$].
- Bracket notation [e.g.: (+2) + (-3) = -1]

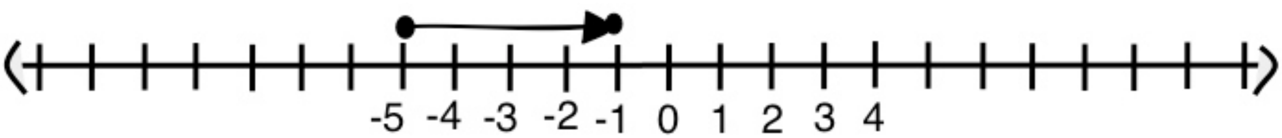
The lesson on integer addition began with bracket notation and then shifted to flat notation. This lesson on integer subtraction uses that approach as well because it makes clear the distinction between the integer and the arithmetic operation (subtraction, in this case).

> The following lesson makes use of the take away and comparison meanings of subtraction.
>
> Refer to: The meanings of the arithmetic operations for details.

DEVELOP stage

Activity 1: Addresses achievement indicator 2, 4, and 5 and revisits SET SCENE

Ask a few groups to present their solution. Hopefully, someone will say something like: "*The the temperature warmed up by 4 degrees*." and be able to show that correctly on a number line. If the number line diagram is not forthcoming, you present it (see diagram).



It is important that the two end points (-5 and -1) and the direction of change (in the positive direction: an increase in temperature) is shown on the number line. Ensure students understand the number line representation of the problem.

- Tell students that it would be useful to represent the temperature change with a number sentence. Present a problem concerning comparison subtraction such as: "*John has 20 candies*. *George has 5 candies*. *Who has more candies and how many more*?" Ask students to solve the problem and provide a number sentence that represents the situation. Hopefully, some of them will provide: 20 - 5 = (15). Discuss how the two amounts of candy are compared by subtraction and whether the temperatures at 9 AM and 4 PM are also being compared.
- Ask students whether subtraction could be used to represent the temperature situation. Ensure they realize there are two possibilities: (-1) - (-5) = ? or (-5) - (-1) = ?.
- Discuss (-5) - (-1) by asking them to think of it as 5 negative counters remove 1 negative counter. Ask students the question: <u>How many counters are left and what type are they (positive or negative)</u>? Ensure they realize the answer to the question is negative 4. Ask whether -4 represents the temperature warming up or getting colder. Ensure students realize -4 represents getting colder and that contradicts what happened (namely, the temperature warmed up by 4 degrees).
- Suggest to students that the correct number sentence is (-1) - (-5) = ? and the answer to this must be +4 because the temperature warmed up. Tell them that the lesson is about being able to do such subtractions and that they will return to temperature situations later.
- Tell them that bracket notation will be used at first to help them see what the integer is and what the subtraction is.

Activity 2: Addresses achievement indicators 2.

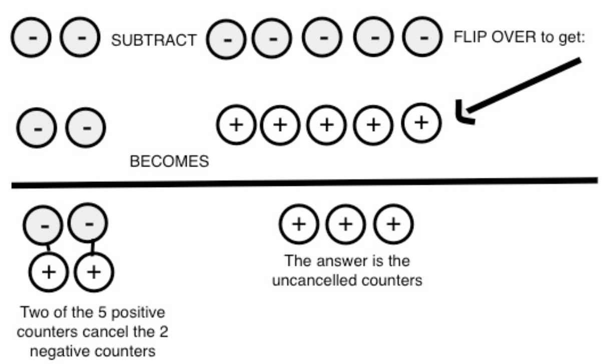
- Ask students to figure out the answer to (+5) - (+2) on a number line. [They will likely start at 5 and walk 2 to the left because that seems the "natural" thing to do.] Ask for solutions. Repeat of the question (+5) - (+4). Ensure all students realize that the sensible thing to do is to: start at the first number and walk to the left.
- Ask them to figure out the answers to (+5) + (+2) & to (+5) + (+4) on a number line (this revisits addition). Ensure they walked to the right, NOT to the left to obtain answers.
- Write (+5) + (+2) and (+5) - (+2) next to each other and discuss the direction walked for the addition question (same as the sign on the 2) and the direction walked for the subtraction question (opposite to the sign on the 2). Repeat this for the questions (+5) + (+4) and (+5) - (+4). Ensure that students realize that they walked in the opposite direction to the sign on the number being subtracted. Ask why that might be the thing to do for subtraction. Ensure students begin to realize that walking in the opposite direction is done because addition and subtraction are opposite operations.

Activity 3: Addresses achievement indicators 2 and 4, and practice.

Provide four integer subtraction questions, one for each case (p - p; p - n; n - n; n - p). Ask students to use a number line to obtain the answers. Ask for and discuss solutions. Ensure students walk the other way when subtracting the second number.

Activity 4: Addresses achievement indicators 4.

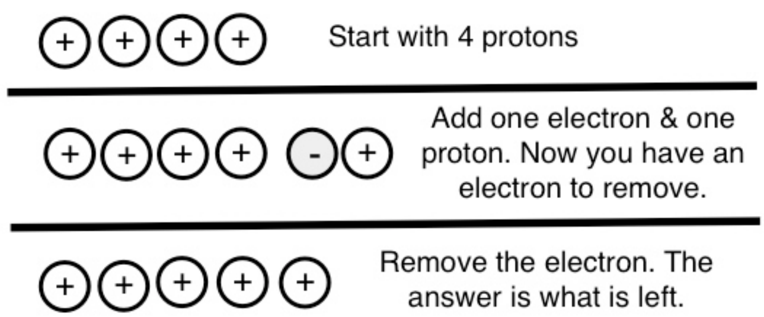
- Provide two-colour counters and establish which colour is positive and which is negative. Present one of the subtraction questions done in activity 3. Suppose the question was: -2 - (-5). Ask students to use the counters to obtain the answer. Suggest that they need to represent both numbers with counters and that they need to "walk the other way" somehow when using the counters. Ask for and discuss solutions. Ensure students realize that "walking the other way" with counters is done by flipping over the counters being subtracted. [See diagram for -2 - (-5) = +3].



- Provide four integer subtraction questions, one for each case (p - p; p - n; n - n; n - p). Ask students to use two-colour counters to obtain the answers. Ask for and discuss solutions. Ensure students represent both numbers of the question and that they flip over the counters when subtracting the second number.

Activity 5: Addresses achievement indicators 4 and 5.

- Provide two-colour counters and establish which colour is positive and which is negative. Tell students that the positive colour represents a proton and the negative colour an electron. [You may want students to research protons and electrons if that was not done for the lesson on integer addition.]
- Tell students to think of subtraction as take away - stress that this means actual removal of things. Tell them that thinking 'take away' also means you DO NOT represent both numbers of a subtraction question, only the first number. Present the following series of subtraction questions: (+4) - (+2); (+4) - (+1); (+4) - (0). Ask students to use the two-colour counters to obtain answers. Ask for and discuss solutions. They should not have difficulty, for example, starting with 4 positives (protons) and the removing 2 positives (protons) to obtain positive 2 (two protons) as the answer.
- Present the question: (+4) - (-1). Ask students to use the two-colour counters to obtain the answer. Now difficulty will emerge. Discuss the fact that there are no negative counters (electrons) to remove. Suggest adding an electron to the "atom". Discuss that the overall charge is now wrong. Ask how to compensate for the added negative charge so that the overall charge returns to what it was before. Ensure students realize that adding a proton (one positive charge) cancels the effect of adding the electron (one negative charge). Ask students whether they now can take way one negative charge (one electron) to get an answer to the question (+4) - (-1). Ensure that students realize that a negative charge is now available to be taken away, and that after doing so, 5 positive charges (five protons) are left. Thus, the answer to (+4) - (-1) is +5. [Refer to the diagram for a picture of the situation and compensation method.]



- Provide a variety of integer subtraction questions that require compensation thinking. Ask for and discuss solutions.

Activity 5: Addresses achievement indicator 5

Note:

By this time, students should have enough experience generating answers to subtraction questions using materials/diagrams and bracket notation (well suited to facilitating understanding). Those experiences would have provided them with answers that make sense to them. Therefore, it is time to develop a short cut (symbolic method) for subtracting integers.

Provide a selected listing of subtraction questions and answers (at least one question for each case) from previous activities. Students should believe the answers because they were obtained using models (number line/counters). Next to each listed subtraction question, write the corresponding addition question that is formed by adding the opposite of the number being subtracted. [Refer to the table for examples.]

CASE	Subtraction Q	Corresponding Addition Q
p - p	(+3) - (+4) = -1	(+3) + (-4) = -1
p - n	(+2) - (-3) = +5	(+2) + (+3) = +5
n - p	(-5) - (+2) = -7	(-5) + (-2) = -7
n - n	(-6) - (-2) = -4	(-6) + (+2) = -4

- ✦ Ask students if they notice anything about the listed subtraction and addition questions that might lead them to a short cut method for subtracting integers. Guide them to realize that subtraction is addition of the opposite. [For example, to do (-2) - (-6), change the subtraction to: (-2) ADD the OPPOSITE of (-6). The subtraction question becomes: (-2) + (+6), and the answer to this is +4.]
- ✦ Provide a variety of integer subtraction questions (written using bracket notation). Ask students to obtain answers using the short cut method. Ask for and discuss solutions.

Activity 6: Addresses achievement indicator 4 and 5.

- ✦ Revisit the SET SCENE subtraction question, (-1) - (-5) = ?, from activity 1. Recall the answer was +4 (the temperature warmed up by 4 degrees). Ask students to use the short cut method to figure out the answer to the question. Ensure they obtain +4.
- ✦ Provide students with the about three temperature problems similar to the SET SECENE one (see below for an example).

  The temperature at 9 AM was 10 Celsius. The temperature at 4 PM was 5 Celsius. What was the change in temperature?

- ✦ Ask them to show the problem on a number line and then solve the problem by "just thinking about" what is going on in it (i.e. Did it warm up or get colder?). Ask them to write the subtraction question [e.g.: Is it (+10) - (+5) or (+5) - (+10)?] that would result in that answer. Ask students to confirm that the answer to the subtraction question matches the answer they figured out by "thinking naturally".

Activity 7: Addresses achievement indicator 5 & SHIFT to flat notation

- ✦ Present four integer subtraction questions, one for each case, using bracket notation. The questions could be: (+5) - (+3); (+5) - (-3); (-5) - (+3); (-5) - (-3). Remind students about the integer notation engineers, scientists, etc. use. Ask students what each number sentence would look like using that notation (give it a name - you could call it science notation). You will have to assist with this task. Ensure they can appropriately write the questions using flat notation [for the examples: 5 - 3; 5 - (-3); -5 - 3; -5 - (-3)]
- ✦ Ask students if the short cut method for subtracting integers they figured out earlier might work for the way of writing integer subtraction that engineers and such people use. Discuss. Do an example of each case with them (e.g. 2 - 3; 2 - (-3), -2 - 3; -2 - (-3) so they realize the short cut method of adding the opposite still works.
- ✦ Discuss that a question such as 4 - 5 can be interpreted as 4 subtract positive 5 or as 4 add negative 5. Ensure students realize each interpretation results in the same answer.
- ✦ Provide students with a mixture of about eight integer subtractions that involve all four cases (p - p, n - n, p - n, n - p) and that use flat notation [e.g. -7 - 8; -6 - (-4)]. Ask students to use the short cut method to figure out the answers.

Activity 8: Addresses achievement indicator 5 & practice.

Organize students into pairs. Each member of a pair makes up 4 integer subtraction questions using flat notation, including the answers. The members figure out the answers to each others questions and correct them.

Activity 9: Assessment of teaching.

Provide students with two integer subtractions using flat notation. One question involves a negative subtract a positive [e.g. -3 - 7]; the other a negative subtract a negative [e.g. -3 - (-7)]. Ask students to:

- determine the answers, using the short cut method
- explain why each answer is correct, using a number line or two-colour counters.

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partially well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheets.

Question 1.

Use a number line to show why each subtraction is true:

a) 2 - (-2) = 4

b) 5 - (-6) = 11

c) -7 - (+2) = -9

Question 2.

Use two-colour counters to show why each subtraction is true:

a) 2 - (-2) = 4

b) 5 - (-6) = 11

c) -7 - (+2) = -9

Question 3.

Use a number line to obtain the answer to each integer subtraction.

a) -2 - 4

b) 4 - (-5)

c) -1 - 3

Question 4.

Change each subtraction question into an addition question that involves adding the opposite.

a) -3 - 4

b) -2 - (-1)

c) 6 - 5

d) 8 - (-2)

Question 5.

Determine the answer to each integer subtraction, using the short cut method.

a) -10 - 3

b) -10 - (-3)

c) 10 - 3

d) 10 - (-3)

MAINTAIN stage

Mini-task example

Every so often:

- Present four integer subtraction questions (one for each case: pos - pos; pos - neg; etc.) Ask students to determine the answers.

Rich-task example

Ask students to solve the following problem:

A polar bear migrates north in the winter time to hunt seals on the sea ice. The daily average temperature decreases 2^{o} Celsius for every 100 km journeyed. At the start of the bear's journey, the temperature is 5^{o}C. What is the temperature after 300 km, after 500 km, after 1000 km?

Comments

This is a rich-task because it is a complex problem that integrates integer subtraction with ratio and science.